

Uncertainty in extreme wind speed estimates: a bootstrap sensitivity study of the FT1 penultimate distribution

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SUMMARY:

This study examines the sensitivity of extreme wind speed estimates to record length when using the three-parameter FT1 penultimate distribution. Ten subsets of growing length (the first 15, 20, ..., 60 years) of wind speed data from Trapani-Birgi airport, Sicily, are fitted with 95% bootstrap confidence intervals (CI) based on 10 000 resamples per subset. Point estimates of the 1000-year wind speed vary by only 5% across record lengths, while the CI width narrows by 32%. Most of this narrowing (85%) occurs within the first 45 years, with diminishing returns thereafter. The shape parameter k stabilises beyond approximately 30 years. The ratio of the CI width at $T = 10000$ years to that at $T = 1000$ years remains approximately 1.40 regardless of record length.

Keywords: extreme value analysis, FT1 penultimate distribution, record length, sensitivity analysis, bootstrap confidence intervals, design wind speed

1. INTRODUCTION

Design wind speeds for structures are obtained by fitting statistical distributions to records of annual maxima and extrapolating to long return periods. The classical Gumbel distribution assumes full convergence to the asymptotic extreme value form, yet wind speed records are rarely long enough for this to hold (Harris, 1996, 2004). The FT1 penultimate distribution (Cook and Harris, 2004) accounts for incomplete convergence by incorporating the parent Weibull shape parameter k , producing curvature on Gumbel probability paper that better represents the tail behaviour of wind speed data. Cook (2004) developed a non-parametric bootstrap method for confidence limits on preconditioned composite FT1 distributions in mixed climates. Using Monte Carlo simulations, Torrielli et al. (2013) found that three-parameter fits are ill-conditioned for records shorter than approximately 50 years. This study applies the FT1 penultimate distribution to ten subsets of growing length (15 to 60 years) from the Trapani-Birgi airport, Sicily, with $B = 10000$ non-parametric bootstrap resamples (Efron and Tibshirani, 1993) per subset, to examine how the fit and its confidence intervals vary with record length.

2. METHODOLOGY

2.1. FT1 Penultimate Distribution and Bootstrap Estimation

The FT1 penultimate cumulative distribution function (Cook and Harris, 2004) extends the Gumbel model through a power-law transformation:

$$F_{\hat{V}}(v) = \exp \left\{ -\exp \left[-a^k (v^k - u^k) \right] \right\} \quad (1)$$

where k is the parent Weibull shape parameter, $a = 1/c$ is the reciprocal of the parent Weibull scale c , and u is the location. For $k = 1$ the model reduces to the Gumbel distribution. The transformation $z = v^k$ makes the reduced variate $y = a^k(v^k - u^k)$ linear in z for fixed k , so that a and u can be estimated by weighted least squares using inverse variance weights from the

exact expectation plotting positions (Harris, 2009). A grid search over $k \in [1.0, 6.0]$ with step $\delta k = 0.001$ selects the value minimising the root mean square error (Harris, 1996).

The design wind speed $\hat{V}_T$ for a return period T is obtained by inverting Eq. (1):

$$\hat{V}_T = \left[u^k + a^{-k} \left(-\ln \left[-\ln \left(1 - \frac{1}{T} \right) \right] \right) \right]^{1/k} \quad (2)$$

Uncertainty in $\hat{V}_T$ is quantified by drawing $B = 10000$ bootstrap resamples with replacement from the N annual maxima (Efron and Tibshirani, 1993). For each resample the full three-parameter model is refitted, including the grid search over k . The corresponding return level $\hat{V}_T$ is then computed via Eq. (2). Re-estimating k at each iteration is essential since fixing it would underestimate the CI width by ignoring uncertainty in the tail curvature. The 95% CI is taken as the 2.5th and 97.5th percentiles of the bootstrap return levels $\hat{V}_T$.

2.2. Sensitivity Analysis Setup

The analysis uses wind speed records from the Trapani-Birgi airport in Sicily (1965–2025, 60 years of data). Three independent annual maxima per year are extracted with a minimum separation of two days, and a lower threshold of 17.5 m/s is applied to isolate the dominant synoptic mechanism from the mixed wind climate of the region. Some yearly maxima fall below this threshold and are discarded, so a year may contribute fewer than three maxima. Ten subsets are formed by retaining the first $L = 15, 20, 25, \dots, 55, 60$ years of the record. All subsets share the same start year (1965), so each smaller subset is fully contained in every larger one. The same 3-parameter fit and bootstrap configuration ($B = 10000$, identical seed) are applied to every subset, so differences across subsets reflect record length, not a change in procedure.

3. RESULTS

3.1. Parameter Convergence and Point Estimate Stability

Table 1 summarises the results for all record lengths. The shape parameter k decreases from 4.255 ($L = 15$ yr) to 2.310 ($L = 60$ yr), settling into the approximate range 2.3–2.8 beyond 30 years (Fig. 1). All fitted values exceed $k = 1.35$ prescribed by the Italian wind loading guideline for Sicily (CNR – Consiglio Nazionale delle Ricerche, 2018), confirming that the three-parameter fit absorbs curvature from the data rather than recovering the true parent distribution parameter (Cook and Harris, 2004).

Table 1. FT1 penultimate shape parameter, design wind speeds, and 95% bootstrap CI widths for all record lengths. N : number of retained annual maxima; W : CI width.

L (yr)	N	k	$\hat{V}_{1000}$ (m/s)	W_{1000} (m/s)	$\hat{V}_{10000}$ (m/s)	W_{10000} (m/s)
15	38	4.255	32.46	15.61	34.37	21.70
20	49	3.311	33.17	15.20	35.59	20.76
25	63	3.124	32.81	14.50	35.29	19.97
30	73	2.720	32.86	14.33	35.60	20.07
35	87	2.524	32.61	13.18	35.45	18.58
40	97	2.536	32.13	12.33	34.90	17.54
45	105	2.838	31.83	11.41	34.37	16.09
50	111	2.818	31.58	10.77	34.12	15.29
55	118	2.532	31.85	10.92	34.63	15.71
60	122	2.310	32.12	10.68	35.13	15.36

Fig. 1 shows all three parameters as a function of L . The shape k (top) drops steeply over the first 30 years before levelling off; the scale a (middle) increases as k decreases, reflecting a trade-off between the two; the location u (bottom) decreases monotonically from 21.57 to 19.39 m/s. Despite these parameter shifts, point estimates are stable: $\hat{V}_{1000}$ varies by only 1.6 m/s (5% of its minimum across record lengths) and $\hat{V}_{10000}$ by 1.5 m/s (4%).

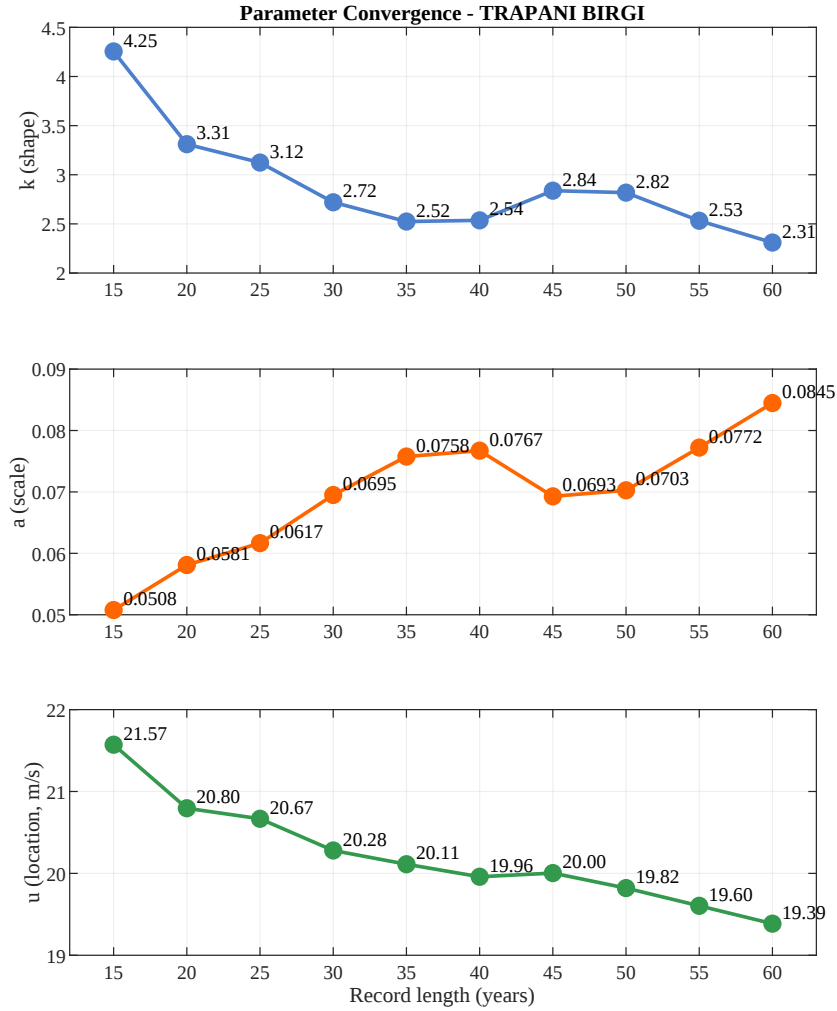


Figure 1. FT1 penultimate parameters as a function of record length L : shape k (top), scale a (middle), and location u (bottom). The shape parameter enters the range 2.3–2.8 beyond approximately 30 years.

3.2. Confidence Interval Reduction

While point estimates change little with record length, the CIs narrow substantially, as shown in Fig. 2. At $T = 1000$ years, the 95% CI width, W_{1000} , decreases from 15.6 m/s ($L = 15$ yr) to 10.7 m/s ($L = 60$ yr), a 32% reduction. At $T = 10000$ years, the decrease is from 21.7 to 15.4 m/s (29%). The ratio W_{10000}/W_{1000} remains between 1.37 and 1.44 across all record lengths, indicating that the CI at $T = 10000$ is consistently about 40% wider than at $T = 1000$, regardless of sample size. The rate of improvement exhibits diminishing returns. Extending the record from 15 to 45 years captures 85% of the total reduction in W_{1000} , while the final 15 years (45 to 60 yr) contribute only 15%, qualitatively consistent with $1/\sqrt{N}$ scaling.

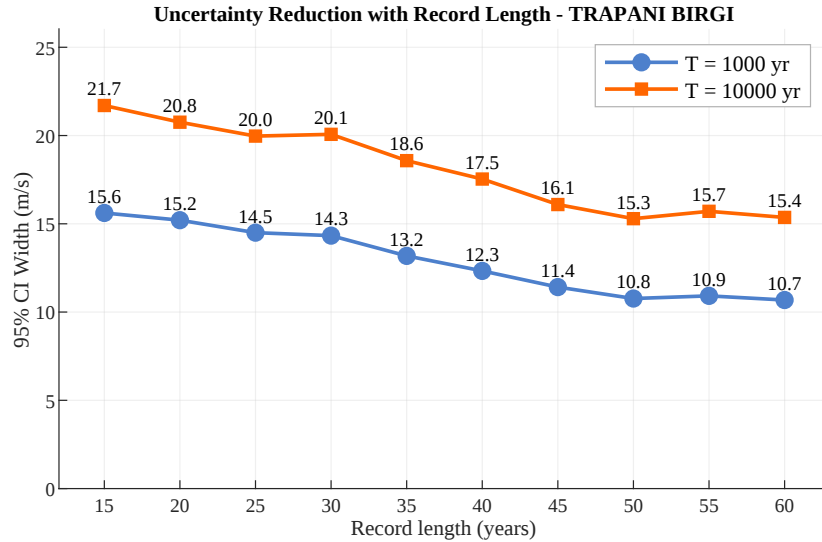


Figure 2. 95% CI width as a function of record length L for $T = 1000$ years (blue) and $T = 10000$ years (orange). Both curves decrease monotonically with a diminishing rate of improvement.

4. CONCLUSIONS

Four main findings emerge from this analysis. First, the shape parameter k is the most sensitive to record length, requiring approximately 30 years to stabilise; point estimates of $\hat{V}_{1000}$, by contrast, vary by only 5% even for records as short as 15 years. Second, the 95% CI width at $T = 1000$ years narrows by 32% from 15 to 60 years, but 85% of this narrowing occurs within the first 45 years. Third, $W_{10000}/W_{1000} \approx 1.40$ across all record lengths. Fourth, even 60 years of data yields a CI width of $\pm 22\%$ of the point estimate at $T = 10000$ years. The analysis is being extended to additional Italian stations, including Genoa, Milan, and Rome, and results will be presented at the conference.

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