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Combined influence of oncoming turbulence on vortex shedding dynamics

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SUMMARY:
Vortex-induced vibrations (VIV) are typically modeled under smooth-flow assumptions, whereas real atmospheric conditions involve turbulence whose effects remain poorly understood. This study investigates the influence of along-wind and cross-wind turbulence on wake dynamics by extending a classical wake-oscillator model. A stochastic formulation is developed by introducing turbulence as quasi-steady perturbations under a small-intensity assumption. The resulting nonlinear stochastic system is reduced to slow amplitude-phase dynamics using both multiple-scale analysis and stochastic averaging, and the predictions are validated against Monte Carlo simulations. The resulting model reveals that turbulence primarily affects phase dynamics through along-wind fluctuations, while amplitude dynamics remains largely unchanged and cross-wind turbulence has limited influence.

Keywords: vortex-induced vibrations, wake oscillator, turbulence, nonlinear stochastic dynamics, perturbation methods

1. INTRODUCTION

Vortex-induced vibrations have long presented challenges to engineers. It is usually modelled with the spectral approach developed by Vickery and coworkers, or with the wake-oscillator approach under the assumption of smooth oncoming flows initially promoted by the works of Hartlen and Currie. Due to the complex interactions between stochastic turbulence and non-linear wake dynamics, rigorous mathematical modeling of VIV under turbulent inflow remains significantly more challenging than in the smooth-flow case. Despite the limited availability of fully developed stochastic VIV models, valuable qualitative insights can still be gained by incorporating stochastic perturbations into existing deterministic frameworks (Denoël, 2020). In this study a randomized version of the latter is considered, aiming at studying in an alternative manner the influence of both the along-wind and cross-wind turbulence in the oncoming flows on the dynamics of vortex shedding.

Despite extensive experimental studies on turbulence's impact on VIV, no clear consensus has emerged: showing suppression, amplification, or negligible influence. These conflicting results highlight the complexity of the problem. Additionally, in contrast with the linear stochastic dynamics problem, the time varying length of the lamina leads the system to be non-linear, and the responses to be non-Gaussian. The stochastic analysis of such systems is particularly difficult because of these "non's" (Zhu et al., 2001).

In this study, a randomized wake equation is first developed by introducing turbulence as quasi-steady perturbations under a small-intensity assumption. The resulting nonlinear stochastic system is then reformulated in terms of slow amplitude and phase variables, enabling the derivation of reduced-order dynamics. Two complementary approaches, multiple-scale analysis and stochastic averaging, are employed and systematically compared to assess their ability to capture turbulence-induced effects. The theoretical predictions are subsequently validated against Monte Carlo simulations under realistic turbulent inflow conditions.

2. PERTURBED WAKE OSCILLATOR MODEL

The present study adopts the wake-oscillator model proposed by (Tamura and Matsui, 1980), focusing only on the wake dynamics by considering a fixed cylinder configuration. A stochastic extension of the wake equation is introduced by randomizing model parameters under a quasi-steady assumption, following the approach of (Mannini, 2020). The governing equation of the wake oscillator for the fixed cylinder read as:

$$\ddot{\alpha} - 2\varepsilon\omega_v(1 - \frac{4f_m^2}{C_{L0}^2}\alpha^2)\dot{\alpha} + \omega_v^2\alpha = 0 \tag{1}$$

where α denotes the state variable of the wake oscillator, the small parameter ε characterises the nonlinear damping, and ω_v is the vortex shedding frequency. The coefficients f_m and C_{L0} correspond to the Magnus effect factor and lift coefficient amplitude, respectively.

Owing to the separation of time scales, where turbulence evolves one to several orders of magnitude more slowly than the intrinsic wake oscillations, the along-wind and cross-wind turbulent fluctuations, u and w, are introduced as quasi-steady perturbations in the present formulation. Specifically, turbulence is incorporated into the shedding frequency ω_v by replacing the mean wind speed U_∞ with the instantaneous one $\sqrt{(U_\infty + u)^2 + w^2}$. In addition, the relative angle of attack of the wake oscillator is perturbed through a modification of $-w/(U_\infty + u)$ in the restoring moment term. Assuming small turbulence intensity, a first-order expansion is retained, yielding the following stochastic wake-oscillator equation, which combines the two types of turbulence considered separately in (Denoël, 2020) and (Rigo et al., 2022).

$$\alpha'' - 2\varepsilon(1 - \gamma\alpha^2)\alpha' + (1 + 2\varepsilon I_{u0}\eta_u)\alpha = \varepsilon I_{w0}\eta_w \tag{2}$$

where primes denote differentiation with respect to the dimensionless time $\tau = \omega_v t$. The coefficient $\gamma = 4f_m^2/C_{L0}^2$ is a constant, while I_{u0} and I_{w0} represent the along-wind and cross-wind turbulence intensities scaled by ε (hence of order one). The stochastic processes $\eta_u(\tau)$ and $\eta_w(\tau)$ denote the normalized along-wind and cross-wind turbulent fluctuations, respectively.

Eq. (2) constitutes a nonlinear stochastic differential equation where the nonlinear term arises from the self-excited nature of the wake oscillator, while the along-wind turbulence introduces a parametric excitation, and the cross-wind turbulence acts as an additive stochastic forcing. The coexistence of these terms makes the analysis of the system particularly challenging.

3. STOCHASTIC SLOW DYNAMICS MODEL

Given that the turbulence intensities are relatively small and that the limit cycle of the unperturbed wake oscillator is stable, the stochastic perturbations are expected to induce only small deviations from the original periodic orbit: perturbations in the transverse direction to the limit cycle remain limited, whereas perturbations along the cycle can become significant over time. This observation suggests that a more appropriate description of the system is obtained by reformulating it in terms of slow amplitude and phase variables. The presence of the small parameter ε enables the use of perturbation techniques to derive equations governing these slow dynamics. In this study, two fundamental approaches, namely multiple-scale analysis and the averaging method, are employed and compared to assess their capability to capture the respective effects of along-wind and cross-wind turbulence.

3.1. Multiple-scale analysis

To apply multiple-scale method, a fast time scale τ and a slow time scale $T = \varepsilon\tau$ are introduced and treated as independent. The solution $\alpha(\tau, \varepsilon)$ of Eq. (2) is then expanded as a perturbation series in ε and dependent both on t and T:

$$\alpha(\tau;\varepsilon) = \alpha_0(\tau, T) + \varepsilon\alpha_1(\tau, T) + \cdots \tag{3}$$

Substituting Eq. (3) into Eq. (2) and collecting terms at each order in ε yield the leading-order solution:

$$\alpha_0 = R(T)\cos[\tau + \phi(T)] \tag{4}$$

where $R(T)$ and $\phi(T)$ denote the slow amplitude and phase, respectively. The perturbed equations at the first order of ε can be then obtained as:

$$\begin{aligned}\partial_\tau^2\alpha_1 + \alpha_1 =& (2\partial_T R - 2R + \frac{1}{2}\gamma R^3)\sin(\tau+\phi) + (2R\partial_T\phi - 2I_{u0}\eta_u R)\cos(\tau+\phi) \\ &+ \frac{1}{2}\gamma R^3\sin(3\tau + 3\phi) + I_{w0}\eta_w\end{aligned} \tag{5}$$

Owing to the low characteristic frequency of the along-wind turbulence relative to the fast time scale τ, the process η_u can be regarded as quasi-constant over one oscillation period. Consequently, the resonant terms proportional to $\sin(\tau+\phi)$ and $\cos(\tau+\phi)$ would induce secular growth in α_1, which can be eliminated by imposing the secularity conditions, yielding the equations governing amplitude and phase:

$$\frac{\partial}{\partial T}\begin{bmatrix} R \\ \phi \end{bmatrix} = \begin{bmatrix} R - \frac{1}{4}\gamma R^3 \\ I_{u0}\eta_u \end{bmatrix} \tag{6}$$

At leading order, the turbulence influences only the phase dynamics but not the amplitude. In particular, only the along-wind turbulence contributes to phase modulation, whereas the cross-wind turbulence does not appear in the reduced slow dynamics. This result raises questions regarding the suitability of the multiple-scale method for stochastic systems. Since the method is inherently developed for deterministic perturbations, its ability to capture stochastic effects relies critically on the assumption of sufficiently slow variations in the stochastic forcing. The absence of cross-wind turbulence in Eq. (6) therefore indicates a potential limitation of the multiple-scale approach in representing the full stochastic dynamics of the wake oscillator.

3.2. Stochastic averaging method

To assess the validity of the multiple-scale analysis, the stochastic averaging method is also employed. This approach is specifically formulated for nonlinear dynamical systems driven by stochastic excitations. It operates by averaging out the fast oscillatory components with respect to an invariant measure, thereby yielding a reduced Markov process governing the slow dynamics. Since the stochastic averaging framework is naturally expressed in terms of stochastic differential equations (SDEs) driven by white noise, the colored processes η_u and η_w are approximated by two independent Ornstein-Uhlenbeck processes, satisfing the following SDEs:

$$\begin{cases} \mathrm{d}\eta_u = -\lambda_u\eta_u + \sqrt{2\lambda_u}\mathrm{d}W_u \\ \mathrm{d}\eta_w = -\lambda_w\eta_w + \sqrt{2\lambda_w}\mathrm{d}W_w \end{cases} \tag{7}$$

where λ_u and λ_w are positive coefficients, and W_u, W_w are standard Wiener processes.

To separate slow and fast dynamics, variables α and α' are transformed into amplitude-phase coordinates via $\alpha = R\cos(\tau+\phi)$ and $\alpha' = -R\sin(\tau+\phi)$. This leads to an augmented 4-DOF system with state vector $\mathbf{X} = (R, \phi, \eta_u, \eta_w)$. According to the stochastic averaging principle, $\mathbf{X}$ converges weakly, as $\varepsilon \to 0$, to a Markov process governed by the following Itô's SDE:

$$\mathrm{d}\begin{bmatrix} \tilde{R} \\ \tilde{\phi} \\ \tilde{\eta}_u \\ \tilde{\eta}_w \end{bmatrix} = \begin{bmatrix} \varepsilon(\tilde{R} - \frac{1}{4}\gamma\tilde{R}^3) \\ \varepsilon I_{u0}\tilde{\eta}_u \\ -\lambda_u\tilde{\eta}_u \\ -\lambda_w\tilde{\eta}_w \end{bmatrix} \mathrm{d}\tau + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \sqrt{2\lambda_u} & 0 \\ 0 & \sqrt{2\lambda_w} \end{bmatrix} \mathrm{d}\begin{bmatrix} W_u \\ W_w \end{bmatrix} \tag{8}$$

The first two equations in Eq. (8) describe the slow evolution of amplitude and phase, which are consistent with the reduced system obtained via multiple-scale analysis (Eq. (6)), up to a scaling factor ε arising from the different time parametrizations. Notably, both approaches indicate that the amplitude dynamics remains unaffected by turbulence at leading order, while the phase dynamics is modulated exclusively by along-wind fluctuations. The absence of cross-wind turbulence in the reduced dynamics therefore appears not to be a methodological artifact, but rather an intrinsic feature of the wake oscillator in response to different turbulence sources.

4. RESULTS

The proposed slow dynamics model is validated against Monte Carlo simulations (MCS) performed directly on the stochastic wake-oscillator equation under turbulent inflow conditions. The turbulence components are generated according to the Kaimal spectrum. Fig. 1 compares the phase-plane trajectories and root-mean-square (RMS) responses obtained from the reduced model and from MCS. The results show good agreement between the two approaches, indicating that the model accurately captures the essential dynamics of the stochastic wake response.

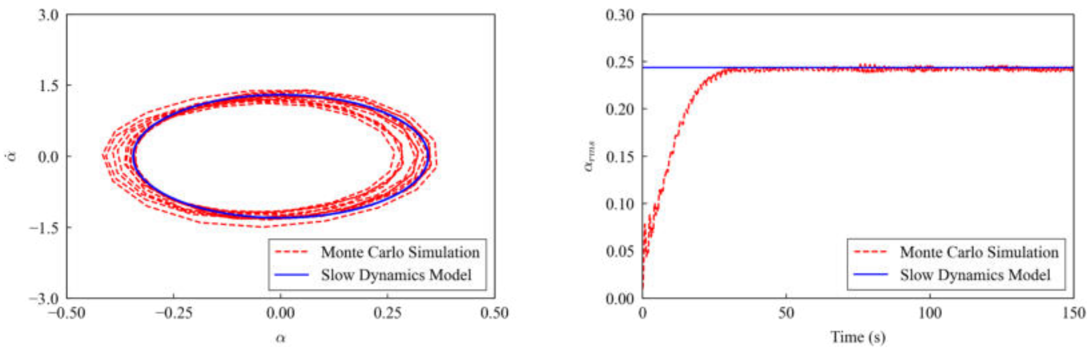

Figure 1. Phase plane (left) and response RMS (right) given by the reduced model and Monte Carlo simulations.

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