



Modal response of a tapered lightning tower subjected to vortex-induced vibrations

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SUMMARY:

This study experimentally investigates vortex-induced vibrations (VIV) on a cylinder and a tapered cylinder model of a lightning rod mast with high slenderness. The cylindrical model exhibits a narrow, well-defined VIV peak, while the tapered cylinder displays multiple broad peaks due to the variation of diameter along the body, leading to an extended lock-in range. The lower-order modes contribute marginally to the response, while higher-order modes dominate the dynamics. Ramp up and ramp down tests showed no hysteresis behaviour for both structures. These findings highlight potential limitations of Eurocode approaches that focus mainly on the first mode. Future work will include experimental testing of intermediate geometries of the mast and enhanced instrumentation to provide detailed measurements and further validate the observed trends.

Keywords: Vortex-induced vibrations, Experimental testing, Tapered structure, Wind tunnel

1. INTRODUCTION

Many civil engineering structures experience vortex-induced vibrations (VIV) during their life-time. Although these vibrations are typically limited in amplitude, they can generate significant cyclic loads and lead to potential fatigue damage or structural failure. This phenomenon has been observed in several lightning rods mounted on a tapered mast. The high-voltage network operator reported fractures at the base of the rods and even complete detachment in some cases.

The VIV response of the structure was evaluated using the Eurocode formulations. An experimental campaign was conducted in parallel to either rebut or confirm the results provided by the current norms. The structure investigated in this study consists of a slender lightning rod approximately 4 meters in length, mounted on a 26-meter cantilever mast shaped as a tapered cylinder. The mast is hollow, with a diameter that varies from 545 to 140 mm, while the rod has a diameter of 40 mm. The deformations are mostly concentrated in the rod which is highly flexible compared to the mast, resulting in a *whip effect* that can induce fatigue or failure. The experimental study focuses on the mast, represented by a standing tapered cylinder or by geometrical variations derived from this configuration.

2. WIND TUNNEL TESTS

The two structures tested in the wind tunnel of the University of Liège are scaled-down models of the mast: a standing 3D-printed tapered cylinder and a standing PVC cylinder with a H/ϕ ratio equivalent to the tapered cylinder (see characteristics in Table 1). The slenderness and small diameter of the models limit the number of sensors that can be installed either on the structure itself or in its wake. At present, measurements have been carried out exclusively using an *ATI Gamma Multi-Axis* force sensor, which records all components of force and torque at the base of the model. Future experiments will include additional measurement techniques,

such as laser-based systems for high-precision displacement measurements at the top of the structure and fibre-optic sensors to capture distributed displacements along the height of the model. The models were tested under uniform flow conditions with flow speeds ranging from 1 to 20 m/s. A ramp up and ramp down procedure on the speed showed that neither model displayed hysteretic behaviour, indicating that the response depends primarily on instantaneous flow–structure interactions.

Table 1. Characteristics of the two models tested in the wind tunnel

	Height H [mm]	ϕ_{bottom} [mm]	ϕ_{top} [mm]	H/ϕ [-]	Taper Ratio T_R	Material
Cylinder	1000	24	24	41.6	-	PVC
Tapered Cylinder	840	20.2	5	41.6 - 168	55.66	PLA

3. RESULTS AND DISCUSSION

The power spectral densities (PSDs) of all signals obtained at different flow speeds can be assembled into a single spectrogram, as represented in Fig. 1. The colour intensity reflects the magnitude of the response, with darker regions indicating higher amplitudes. Dashed lines are superimposed to highlight the main trends of the spectrogram. The eigenfrequencies of the structure are readily identified, as their corresponding spectral peaks align across all flow conditions. In addition, a distinct line corresponding to the forced response induced by vortex shedding is appearing as well. Unlike the eigenfrequency lines, this shedding line is not aligned at a single frequency, as the shedding frequency increases with the flow velocity. When this line intersects an eigenfrequency line, resonance occurs, leading to a significant amplification of the VIV response.

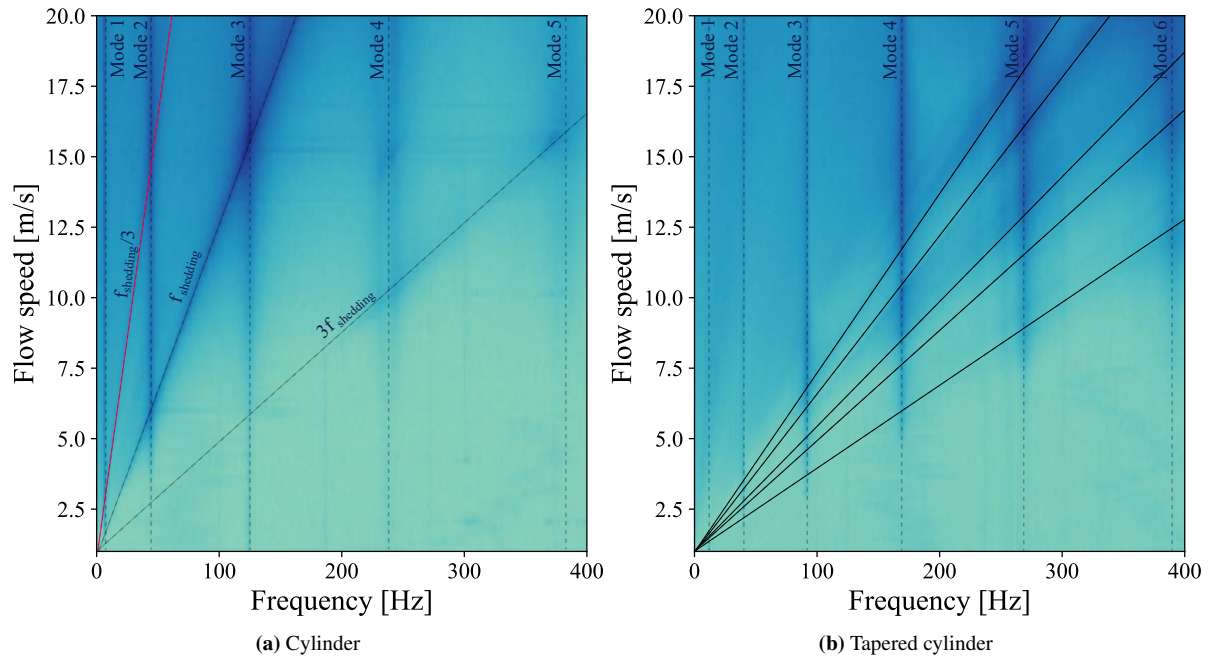


Figure 1. Spectrogram of the PSDs of the cross-flow force at the base of the model with respect to the flow speed, for a cylinder (left) and a tapered cylinder (right). The intensity increases with the darkening of the colour.

The darkest diagonal line in Fig. 1a, labelled $f_{shedding}$, represents the primary vortex shedding frequency of the cylindrical mast. Peak VIV response is expected when this line intersects with an eigenfrequency line. The slope of this line confirms a Strouhal number of approximately

0.18, consistent with the standard value for a circular cylinder, as computed in Eq. (1). Since vortex shedding is not perfectly harmonic, higher harmonics can also interact with the structure. In particular, the second diagonal line, $3f_{shedding}$, observed in Fig. 1a corresponds to the third harmonic of the shedding frequency, which contributes to the excitation of the structure as well.

$$St = \frac{f\phi}{u} = \frac{120.5 \cdot 24 \cdot 10^{-3}}{15.6} = 0.185 \quad (1)$$

The tapered cylinder exhibits a more complex response than the uniform cylinder. The variation in diameter along the structure induces a distribution of local shedding frequencies, enabling excitation at multiple frequencies. Multiple flow velocities can thus satisfy resonance conditions, leading to repeated crossings between the shedding line and the eigenfrequency lines (Gaster, 1969). The spectrogram becomes more diffusive and is characterized by several regions of elevated response amplitude that can be associated with *shedding cells* (Piccirillo and Van Atta, 1993).

The response can be further analysed by bandpass filtering the time responses around each structural frequency to extract the components of the response in each vibration mode as a function of flow velocity, as presented in Fig. 2. The peaks identified correspond to the flow velocities where the shedding line intersects with the eigenfrequency lines in Fig. 1.

The magnitude of these peaks correlates with the intensity observed in the spectrogram. At low frequencies, the response remains weak, consistent with the low contrast observed in the corresponding spectral regions. For the cylindrical case, the strongest response is observed in mode no.3, around 120 Hz. Additional peaks associated with higher harmonics of the shedding frequency are also present and could be attributed to inertial effects within the system.

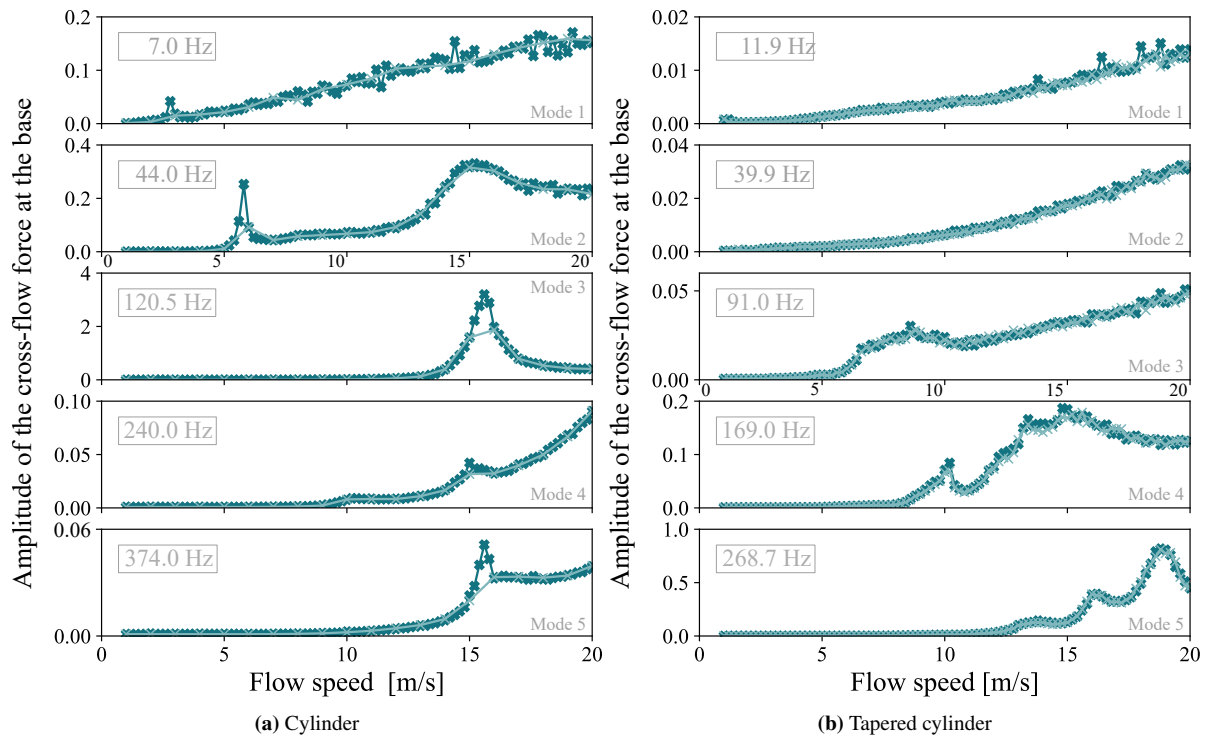


Figure 2. Modal amplitude of the cross-flow force at the base of the model as a function of the flow speed, on a cylinder (left) and tapered cylinder (right) for the ramp up (bold crosses, dark blue) and the ramp down (thin crosses, light blue).

In contrast, the tapered cylinder displays multiple successive peaks in its response. Mode no.4 shows that the wide range of flow velocities associated with vortex shedding results in a broad response without a clearly defined peak. However, mode no.5 exhibits a series of distinct peaks, which may be connected to localized shedding frequencies corresponding to diameters at modal antinodes (Brusco et al., 2023; Visscher et al., 2011).

These results can be interpreted in terms of lock-in, during which the vortex shedding frequency synchronizes with a structural natural frequency. For the cylinder, lock-in occurs over a narrow range of flow velocities, producing sharp and well-defined peaks. In contrast, the tapered cylinder, due to its continuously varying diameter, experiences synchronization over a wider range of flow velocities and at multiple locations along its height. This leads to a broader and more complex VIV response. The amplitude of the response of the tapered cylinder is consistently lower than that of the cylinder at corresponding eigenfrequencies. This difference can be attributed to the diffusive nature of the response, as the vibrational energy is spread over a broader range of flow velocities.

4. CONCLUSION

The present study highlights the fundamental differences in VIV response between a uniform cylindrical structure and a non-uniform structure with a continuously varying diameter. The cylindrical model exhibits a well-defined VIV behaviour, characterized by narrow lock-in regions and distinct resonance peaks associated with clear intersections between vortex shedding and structural eigenfrequencies.

In contrast, the tapered cylinder displays a broad and complex response. The continuous variation in diameter leads to a distribution of local shedding frequencies along the span. This results in a diffusive spectrogram and multiple closely spaced excitation regions, or *shedding cells*. This behaviour produces an extended lock-in range. However, the amplitude of the response is significantly lower compared to the cylinder as the energy is distributed over a wider range of flow velocities.

The analysis demonstrates that higher-order modes can dominate the structural response, while the first mode remains comparatively weak. This finding emphasizes the importance of considering higher modes in the assessment of VIV fatigue and structural integrity. To further support these findings, future work will investigate intermediate geometrical configurations between the cylinder and the tapered cylinder to better characterize the transition in VIV behaviour. Additional experimental campaigns, supported by enhanced instrumentation, will also be conducted to provide detailed insight into the spatial distribution of the structural response.

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